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Choquet Integral as an Alternative Aggregation Method to Measure the Overall Academic Performance of Primary School Students: A Case Study

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Abstract. Many average methods are available to aggregate a set of numbers to become single number. However these methods do not consider the interdependencies between the criteria of the related numbers. This paper is highlighting the Choquet Integral method as an alternative aggregation method where the interdependency estimates between the criteria are comprised in the aggregation process. The interdependency values can be estimated by using lambda fuzzy measure method. By considering the interdependencies or interaction between the criteria, the resulted aggregated values are more meaningful as compared to the ones obtained by normal average methods. The application of the Choquet Integral is illustrated in a case study of finding the overall academic achievement of year six pupils in a selected primary school in a northern state of Malaysia.

Keywords: academic achievement, aggregation, average, Choquet Integral, interdependencies
PACS: 1.40Gm; 02.30.Rz; 02.30.Cj

INTRODUCTION

Aggregation is basically a process of combining a set of numbers to become a single number. The simple arithmetic average (SA) is the most popular aggregation method due to its simple feature which does not involve complex calculation. By using SA, the final score is obtained by adding all values and divide the total with the number of related values. For example, if a student has five scores from five different tests, his/her final overall score is determined by summing all the five scores and divide the sum by five.

Simple weighted average (SWA) is another aggregation method that assigns different values which are called as weights to the values or the criteria of values considered. These weights may represent the relative importance [1] of the criteria. These weights react as the coefficients of the values since each weight is multiplied to its corresponding value and total up these products to become the final overall score. In the case of SA method, equal weights are given to the criteria since in the case of calculating the average of five scores, the calculation is done by multiplying each score with one fifth or 0.2, and add all the products. Equal weights can be interpreted as the criteria are of equal importance, which are not really true in the real application [2], since evaluation criteria in most cases are of different important. Hence SWA with different weights values is a better method than SA because SWA can give more meaningful interpretation to the aggregation process when it is used in solving real problem [3]. However the issue of interdependency or interrelation between the criteria is still being ignored in both methods.

The aim of this paper is to introduce a powerful method called Choquet aggregation method or Choquet Integral [4, 5], which considers the interactions or interdependencies in the aggregation process. These interdependency values are predetermined before the aggregation process is able to be carried out. This paper illustrates the use of lamda (λ) fuzzy measure method [6] to determine the interdependencies of criteria in the selected case study. The 2011 mid-semester examination overall performance of 33 year six pupils in a high performance primary school in the State of Perlis, Malaysia was recalculated by Choquet Integral. The case study is used as a basis of discussion on how Choquet Integral is benefited.

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CHOQUET INTEGRAL

As mentioned earlier, most of the traditional aggregation techniques such as SA, SWA, Geometric Average or Geometric Weighted Average methods are treating the criteria or attributes independently. For example, if any one of these methods is used in finding the overall academic performance of students, the interactions between the academic subjects are ignored. This implies that the students' performance in one subject does not have any relation with their performance in other subjects which is generally not true in many situations. However, Choquet integral plays an important role in capturing the interaction aspect among criteria during aggregation process [7]. Besides that, Choquet integral has the merit in producing unique solution in comparison to the other aggregation techniques [8]. Choquet Integral is a non-additive aggregation method which has the ability to model issues of dependencies between criteria. So, this model can be used in non-linear situations since it does not need to assume the independence of each criterion. There are three basic steps to apply Choquet integral in solving MCDM problem [9]. First, the problem must be well-defined and the relevant criteria must be determined. Then, the fuzzy measure weights are estimated. Finally, the overall scores of each alternative will be computed when the identified fuzzy measure weights are applied into Choquet integral model to compute the overall scores.

METHODOLOGY

This section has three subsections which explain about the case study, the method used to estimate weights of the academic subjects, the λ-fuzzy measure method and the Choquet integral method. The weights of the criteria have to be determined first before the λ-fuzzy measure can be utilized to estimate the interdependencies between criteria. Then, once the interdependency estimates are available, the Choquet Integral is ready to be used to aggregate the students' individual achievements.

Case Study

All year-six pupils in Malaysia would sit a national examination which is called as *Ujian Penilaian Sekolah Rendah* (*UPSR*) or Primary School Evaluation Test (PSET) at the end of the year. The test consists of five subjects: Mathematics (Math), Science, Malay Language Comprehension (Malay Comp), Malay Language Written (Malay Written) and English Language (Eng), where the scores are standardized in the range of 0 to 100. All primary schools would conduct alike tests as preparation for their pupils towards UPSR since the results of UPSR become a basis for entrance to boarding schools in Malaysia. The results from the school's level tests are usually used by the teachers to monitor the preparedness of their pupils towards UPSR. In this case study, the 2011 mid semester results of an internal examination for 33 year-six students of one high performance primary school in state of Perlis, Malaysia was chosen as the case study. This school was selected due to its recognition and its teachers were working hard to sustain the standard of the school.

Weighting Method

In general, the weighting methods can be classified into two main approaches, subjective and objective approaches [10]. The objective approach depends heavily on the quantitative intrinsic information contained in each criterion, where the information is manipulated mathematically to generate new information such as the standard deviation, entropy, correlation, and variation coefficient [11, 12, 13]. The subjective approach requires evaluator(s) to evaluate the criteria in terms of the relative importance or influence of the criteria towards final score. Many methods are available such as Analytical Hierarchy Process [14] and rank-based method [15]. In this study, the subjective method, called the direct rating method [16, 17, 18] was used to estimate the relative importance of the five subjects where 5 teachers were asked to evaluate academic subject by using linguistic scales as in the following table.

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TABLE (1): Linguistic scales for the importance weight

The subject is less important	Extremely	0.0
	Highly	0.1
	Very	0.2
	Strongly	0.3
	Moderately	0.4
	Equally	0.5
The subject is more important	Moderately	0.6
	Strongly	0.7
	Very	0.8
	Highly	0.9
	Extremely	1.0

λ-Fuzzy Measure

This measure which has been introduced by Sugeno in 1974, appears to be widely used due to its ease of usage, mathematical soundness and modest degree of freedom properties. Let $X = \{x, \dots, x_n\}$ be the set of academic subjects and $P(X)$ denotes the power set of X or set of all subsets of X. A fuzzy measure g on the set of subjects is a set function $g_\lambda: P(X) \rightarrow [0, 1]$ satisfying certain properties such as boundary condition, monotonicity, super-additive and sub-additive [19]. However, since g is non-additive in general, it is necessary to define the 2^n coefficients corresponding to the 2^n subsets of X. The fuzzy measure of a finite set can be obtained from the set of values of individual weights of the academic subjects or the fuzzy density $g_i = g_\lambda(\{x_i\})$ for $i = 1,2,\dots,n$ which can be formulated as the following equation.

$$g_\lambda(\{x_1, x_2, \dots, x_n\}) = \frac{1}{\lambda}\left|\prod_{i=1}^{n}(1+\lambda g_i) - 1\right| \tag{1}$$

Based on (1), and since the boundary conditions, $g_\lambda(X) = 1$, the parameter λ, where $-1 \le \lambda < 0$ can be calculated by solving the following equation.

$$\lambda + 1 = \prod_{i=1}^{n}(1 + \lambda \cdot g_i) \tag{2}$$

Choquet Integral Formula

The general Choquet Integral formula can be stated as in the following equation.

$$(C)\int f \;\; dg = f(x_n)g_\lambda(F_n) + [f(x_{n-1}) - f(x_n)]g_\lambda(F_{n-1}) + \dots + [f(x_1) - f(x_2)]g_\lambda(F_1) \tag{3}$$

where $f(x_1) \ge f(x_2) \ge \dots \ge f(x_n)$, $F_1 = \{x_1\}$, $F_2 = \{x_1, x_2\}, \dots, F_n = \{x_1, x_2, \dots, x_n\} = X$

RESULT AND DISCUSSION

This section is divided into 4 subsections which provide summary of raw data, the individual fuzzy density or weights of individual academic subjects, the interdependency values of all subsets of academic subjects and the overall scores of the students using two different aggregation methods.

Summary of Raw Data

The summary of the raw marks of the 33 students is portrayed in Table 2.

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TABLE (2): Summary of raw scores

Subject	Malay Comp	Malay Written	Eng	Math	Science
Maximum	88.0	85.0	80.0	69.0	68.0
Minimum	48.0	46.0	40.0	23.0	28.0
Mean	69.8	68.5	69.4	51.2	50.4
Median	68.0	70.0	70.0	51.0	50.0
Standard deviation	9.4	11.2	7.4	10.4	9.5

Weights of Academic Subjects

Table 3 shows the individual judgments by the selected teachers where T1, T2, T3, T4 and T5 represent teacher 1, 2, 3, 4 and 5 respectively. While the last column shows the final weights of the five subjects obtained as average weights. In general, highest weight is given to Science, followed by Math, Malay Comp and Malay Written have the same weight while English is having the lowest weight value.

TABLE (3): Judgment of the relative importance of five subjects by five experts and the final weights

Subjects	Experts					
	T1	T2	T3	T4	T5	Final Weights
Math	0.50	0.50	0.50	0.60	0.50	0.52
Science	0.50	0.50	0.60	0.60	0.50	0.54
Malay Comp	0.30	0.40	0.50	0.30	0.30	0.36
Malay Written	0.30	0.30	0.40	0.40	0.40	0.36
Eng	0.30	0.30	0.40	0.30	0.40	0.34

Interdependency Values

After the individual weight or the fuzzy density for each academic subject is obtained, the fuzzy measures which represent the degree of interdependence for all subsets of the academic subjects can be calculated by using (2). Since there are five subjects being considered, there are 32 subsets available.

TABLE (4): Identified interdependency estimates between the academic subjects

No.	Sets	Fuzzy measure	No.	Sets	Fuzzy measure
1.	$g_\lambda(\{\phi\})$	0.00	17.	$g_\lambda(\{x_1, x_2, x_3\})$	0.75
2.	$g_\lambda(\{x_1\})$	0.36	18.	$g_\lambda(\{x_1, x_2, x_4\})$	0.83
3.	$g_\lambda(\{x_2\})$	0.36	19.	$g_\lambda(\{x_1, x_2, x_5\})$	0.84
4.	$g_\lambda(\{x_3\})$	0.34	20.	$g_\lambda(\{x_1, x_3, x_4\})$	0.83
5.	$g_\lambda(\{x_4\})$	0.52	21.	$g_\lambda(\{x_1, x_3, x_5\})$	0.84
6.	$g_\lambda(\{x_5\})$	0.54	22.	$g_\lambda(\{x_1, x_4, x_5\})$	0.90
7.	$g_\lambda(\{x_1, x_2\})$	0.60	23.	$g_\lambda(\{x_2, x_3, x_4\})$	0.83
8.	$g_\lambda(\{x_1, x_3\})$	0.59	24.	$g_\lambda(\{x_2, x_3, x_5\})$	0.84
9.	$g_\lambda(\{x_1, x_4\})$	0.71	25.	$g_\lambda(\{x_2, x_4, x_5\})$	0.90
10.	$g_\lambda(\{x_1, x_5\})$	0.72	26.	$g_\lambda(\{x_3, x_4, x_5\})$	0.90
11.	$g_\lambda(\{x_2, x_3\})$	0.59	27.	$g_\lambda(\{x_1, x_2, x_3, x_4\})$	0.91
12.	$g_\lambda(\{x_2, x_4\})$	0.71	28.	$g_\lambda(\{x_1, x_2, x_3, x_5\})$	0.92
13.	$g_\lambda(\{x_2, x_5\})$	0.72	29.	$g_\lambda(\{x_1, x_2, x_4, x_5\})$	0.96
14.	$g_\lambda(\{x_3, x_4\})$	0.70	30.	$g_\lambda(\{x_1, x_3, x_4, x_5\})$	0.96
15.	$g_\lambda(\{x_3, x_5\})$	0.71	31.	$g_\lambda(\{x_2, x_3, x_4, x_5\})$	0.96

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16.	$g_\lambda(\{x_4, x_5\})$	0.80	32.	$g_\lambda(\{x_1, x_2, x_3, x_4, x_5\})$	1.00

Table 4 shows the results of the fuzzy measures for all subsets. In the table, x_1 represents Malay Comp, x_2: Malay Written, x_3: Eng, x_4: Math, while x_5: Science. The fuzzy measures in No.1 to No. 6 are the same as the final weights of the subjects as summarized in the previous table. The interdependency estimates in No. 7 up to No. 16 show the combined fuzzy densities of two subjects or the interdependency scores between two academic subjects. Since Math and Science are having the two highest weights, so it is not a surprise that the interaction or the degree of interdependency between these two subjects is the highest, that is, 0.8. The same pattern can been observed in the interaction between three subjects, in No. 18 to No 25; or interaction between four subjects, in No. 26 up to No. 31, the interdependency values are higher when Math and Science subjects are included. No 1 and No 32 represent the boundary property of fuzzy measures.

Overall Academic Achievement Using Choquet Integral Method

Table 5 shows several final scores of the students computed by SA and Choquet Integral methods. The third column shows the final scores obtained by using SA method and its corresponding ranking is available in column 5, whereas column 4 is the overall scores determined by Choquet Integral method with its ranking is in the last column.

Based on the table, it is noticed that students can have equal final scores when SA method is used, for example in No. 6 and 7, and no 28 and 29. However the use of Choquet Integral does not give any equal final scores. Besides that student 31 is still at top position even though two different techniques were used in finding the overall scores. The same pattern can be seen in the last two positions by which the positions were not affected but with different final scores.

TABLE (5): Comparison of final scores between average method and Choquet Integral method

No	Student name	Final score by SA method	Final score by Choquet integral	Ranking order based on average scores	Ranking order based on Choquet integrated values
1	Student 31	72.4	74.5	1	1
2	Student 13	70.2	73.2	2	3
3	Student 5	69.4	72.7	3	5
4	Student 11	68.6	74.2	4	2
5	Student 27	67.4	72.9	5	4
6	Student 3	67.0	70.0	6	9
7	Student 32	67.0	69.8	6	10
8	Student 2	66.0	69.5	8	11
9	Student 20	66.0	71.0	9	7
10	Student 12	65.8	70.8	10	8
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
28	Student 8	55.6	59.6	28	29
29	Student 25	55.6	58.8	28	30
30	Student 9	54.0	58.2	30	31
31	Student 22	53.8	60.4	31	28
32	Student 14	50.0	55.8	32	32
33	Student 7	49.6	55.4	33	33

Based on the result of the case study, Choquet Integral has at least two advantages over SA method. The final scores of the students are more meaningful because the scores are obtained by considering the interactive measures between the academic subjects. Besides that, the Choquet final scores are unique which leads to unique rankings among the students.

CONCLUSION

This paper is highlighting a promising aggregation method, the Choquet Integral as an alternative method in calculating composite scores of multi criteria problems. Prior to the use of the method, the interaction or interdependency between the criteria or evaluation factors must be identified by many available methods such as

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fuzzy measure methods. The interdependencies can be determined for every possible combination of criteria. Finding the composite final scores by using SA method where the interaction between the criteria were not considered, can give a misleading result since the evaluation criteria are usually not strictly independent and the interdependency values are not reflected in the computation.

Since the performance of students in different academic subjects is somewhat interrelated, students' academic achievement should be determined by using Choquet Integral method. In doing so, the final results can reflect the true performance of the students because the interactive measures among the subjects would be included in the aggregation process. So, it is suggested that the school or the related education authorities should use the Choquet ranking as a reference in monitoring the students' progress. The use of this alternative aggregation method is suitable to be used in solving any other multi criteria problems. However, the main drawback of this aggregation method lies at the stage of finding the interdependencies measures. The number of measures is increasing exponentially as the number of criteria is increasing. One way to overcome this problem is by applying the factor analysis method as suggested by [20], or by considering only limited and most influential criteria.

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